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Mesh orientation and cell size sensitivity in 2D SWE solvers

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ABSTRACT: Regular and irregular mesh solvers for the depth averaged Navier-Stokes equations discretise a 2D domain into cells and faces. Many solution schemes consider the cells and faces to be flat-bottomed and either 'wet' or 'dry' depending on whether the water surface elevation exceeds the cell or cell face elevation. This approach can be a very simplified representation of the terrain or bathymetry within the cell that potentially causes inaccuracies and a dependency on mesh size, and in the case of regular mesh solvers, mesh orientation, and in the case of irregular mesh solvers, mesh design. A more physically accurate representation is to consider 'partially wet cells', that use the underlying topography for each cell and face to build non-linear relationships. In the case of cells, between the cell water volume and the water surface elevation, and in the case of faces, between the water surface elevation and the face flux area. The benefits of 'sub-grid sampling' (SGS) appear significant and far-reaching. For whole-of-catchment models, response times and water retention are more realistic. For deep sided channels that are not mesh aligned, flow patterns have no "saw-tooth" effects and obey Manning's formula. However, most significantly, the sensitivity of results to mesh size is greatly reduced and the sensitivity to mesh orientation is almost eliminated. This conclusion is far-reaching in that regular cartesian mesh solvers can produce the same quality of results as well-designed, high resolution, irregular meshes. The SGS methodology is presented, supported by benchmarking to theory, a flume test and case study observations.

1 INTRODUCTION

Flooding is one of the most destructive natural hazards, affecting over 34 million people and causing US$ 19.7 billion damage globally in 2018 alone (CRED, 2019) and with the impact of climate change and growing populations, flood risk is predicted to increase in many parts of the world (eg Alfieri *et al.,* 2018). Accurately predicting and understanding flood hazard through hydrodynamic modelling is important for the purpose of flood risk management at a range of scales throughout the world.

2-Dimensional (2D) hydrodynamic models are commonly used to simulate the flow of water upon terrestrial surfaces to predict flood inundation. The 2D depth averaged Navier Stokes equations, known as the Shallow Water Equations (SWE), are commonly simulated on either a uniform regular (cartesian) mesh of square cells, or an irregular mesh comprised of cells of varying shape and area, typically triangles and quadrilaterals (Lane, 1998). Traditionally, the approach to specifying the cells' terrain elevations is to take either the elevation at the cell centroid or the average elevation within the cell. The resulting mesh is a series of flat-bottomed cells with linear relationships between water surface elevations and cell water volume (cell water depth multiplied by cell area). Furthermore, connections between adjacent cells and the cell faces are rectangular in shape, with linear relationships between water surface elevation and the face flux area used to convey flow. However, the bathymetric or terrain surface data sets (Digital Elevation Models, DEM) are typically available at much higher resolutions than the 2D SWE mesh resolution.

These traditional first order approaches, like that schematised in Figure 1(a), are adequate if the mesh resolution is sufficiently fine to reasonably represent the underlying terrain. However, where there is significant variation in the terrain within a cell, these approaches result in the loss of potentially useful data and may not accurately represent the level of hydraulic behaviour desired to meet the modelling objectives. One solution is to reduce the cell sizes to better depict the higher resolution terrain data, however the computational requirements, both in terms of memory and time, can become limiting and does not always result in significant improvements in model output accuracy (Bates *et al.*, 1998), possibly due to the representation of sub-grid scale turbulence (Collecutt *et al.*, 2020).

2 SUB-GRID SAMPLING (SGS) APPROACH

A sub-grid sampling (SGS) approach was investigated that utilised the higher resolution DEM data as shown in Figure 1(b). The TUFLOW HPC regular mesh 2D finite volume solver (Collecutt & Syme, 2017) was used for the testing. The underlying sub-grid terrain of the cells was sampled at the same resolution as the DEM to develop a non-linear relationship between the water surface elevation and the cell's volume to describe the cells' storage capacity. The sub-grid terrain was also used to generate a non-linear relationship between the water surface elevation and the cell face area and cell width (or wetted perimeter) to improve the representation of the fluxes across the cell faces as flow is conveyed throughout the model domain. The SGS approach still computes a single water level for each cell, but the computations to determine the cell volume and cell face fluxes utilise the higher resolution terrain data.

3 HYDRAULIC BENCHMARK CASES

The SGS approach has been successfully benchmarked to a range of hydraulic scenarios, with substantial benefits arising. The three cases/observations discussed are:

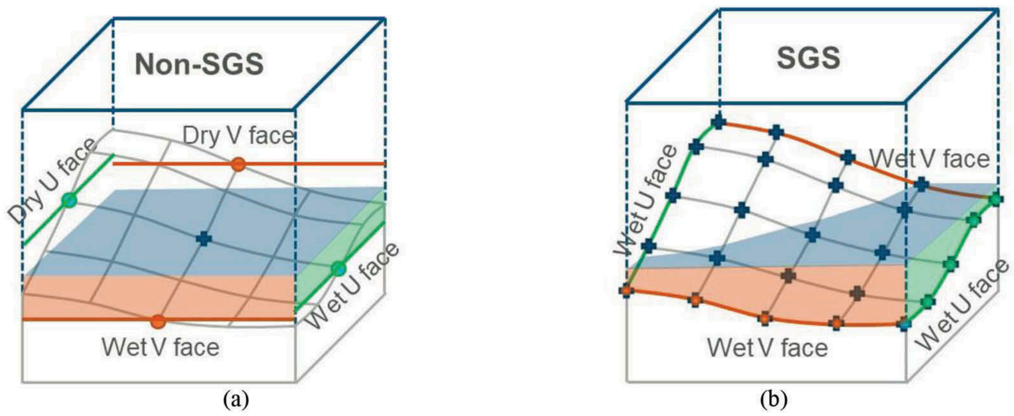

Figure 1. Regular 2D Mesh Terrain Sampling. (a) Traditional approach of a single elevation per cell centre and cell face. (b) Sub-Grid Sampling (SGS) at a higher resolution over the cell and across each cell face. (b) shows how all four cell faces would be active for the same water level compared with two faces for (a).

- Theoretical solution to a rectangular channel.
- Flume test of flow round a smooth bend.
- Applications to real-world models to assess its value in 2D hydrodynamic modelling.

The theoretical case is a 1,000 m long uniform channel with a 100 m wide rectangular section. The test case model represents uniform flow conditions that should reproduce Manning's equation for bed resistance. The computational mesh was rotated to provide various degrees of misalignment with the flow direction, and the simulations compared to the theoretical water level and energy slope derived from Manning's equation.

The flume test is the experimental U-Bend flume (De Vriend, 1978), which comprises a uniform U-bend channel with a shallow rectangular cross-section. During the testing, surface water elevations along the inner bank, centre and outer bank were measured that demonstrated super-elevation effects on the outside of the U-bend. This test introduces 2D in plan effects, whilst the rectangular section theoretical case is primarily a 1D problem. The SGS approach has been applied to a range of existing calibrated models from which observations have been made on the effects of applying SGS. Due to space limitations, a summary of these observations is provided. In each case, the effect of the SGS approach was compared with using the traditional single elevation per cell approach.

4 RESULTS

4.1 *Theoretical rectangular channel*

The theoretical rectangular channel case aptly demonstrates how a regular mesh or poorly designed irregular mesh using a single elevation per cell (Figure 1(a)) fails to reproduce simple uniform flow hydraulics when the wet/dry interface is not aligned to the mesh. This is caused by a flow disturbance created where the water flows into dry or inactive cells along the wet/dry boundary as shown in Figure 2.

Figure 3 shows the water surface gradient along the channel for different orientations of the mesh. The results conform exactly with Manning's equation when the mesh is perfectly aligned with the rectangular channel (0° case). However, as the mesh is rotated the results do not reproduce Manning's equation with an elevated water surface (greater energy loss) occurring. The distorted, non-uniform, velocity field leads to a jagged saw-tooth effect at the wet-dry interface that obstructs the flow field leading to artificial energy losses and the elevated water surface.

However, with SGS applied the results, shown in Figure 4, reproduce Manning's equation for all orientations tested. There is also no distortion of the velocity field with the velocity vectors aligned parallel to the rectangular channel for all tests. These initial results indicate that a regular mesh 2D solver with SGS can accurately reproduce hydraulic flows at any mesh orientation, ie. the 2D solution's results are not dependent on mesh orientation – a powerful proposition for regular (cartesian) mesh 2D models.

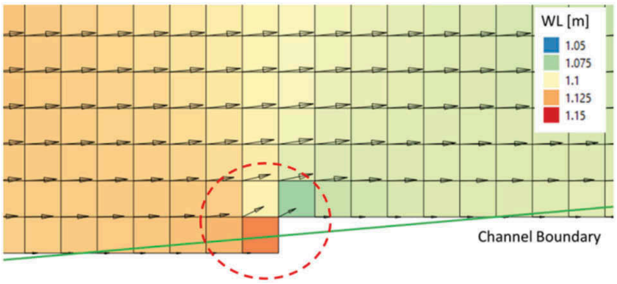

Figure 2. Artefact velocities near wet/dry boundary for single elevation per cell approach.

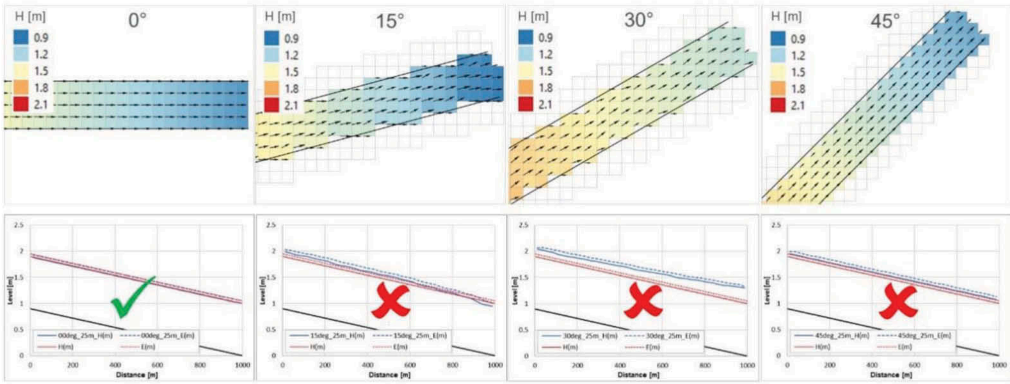

Figure 3. Rectangular uniform channel using traditional single elevation per cell approach for 4 grid alignments (0º, 15º, 30º and 45º). Only 0º conforms to Manning's equation. charts show energy (dashed lines) and water surface (solid lines).

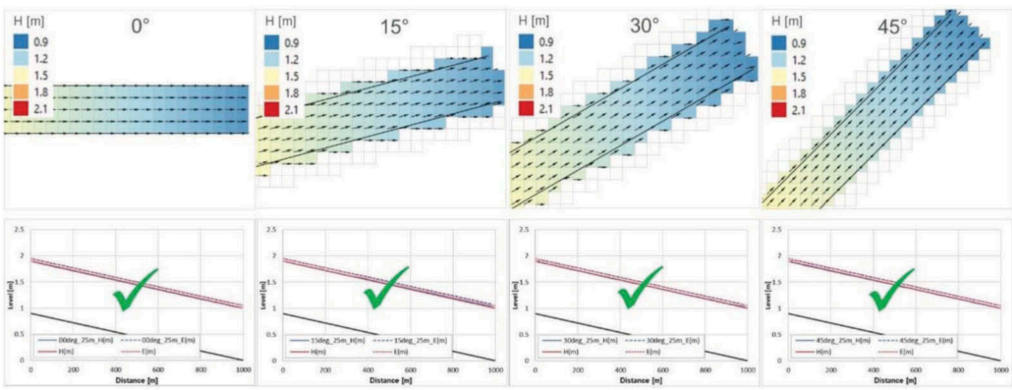

Figure 4. Rectangular Uniform Channel using Sub-Grid Sampling (SGS) Approach for 4 Grid Alignments (0º, 15º, 30º and 45º). Charts show energy (dashed lines) and water surface (solid lines).

For the single elevation per cell approach, the magnitude of the artificial energy losses was also found to be dependent on the number of cells across the channel, as well as the degree of misalignment. (a) shows the results for 5, 10, 25 and 50 m cell sizes using the single elevation approach (a) for the 30° case.

With increasing coarseness (greater cell size) the results have poorer agreement with Manning's equation. This corresponds with 2D modelling guidelines that emphasise the need to have enough cells across the primary flow paths to minimise the sensitivity of a regular mesh's orientation and conformance with Manning's equation (Australian Rainfall and Runoff, 2012). However, the SGS approach, presented in (b), shows excellent convergence to theoretical water level as computed by Manning's equation for the full range of cell sizes. The results suggest that the SGS approach not only eliminates inaccuracies associated mesh orientation, but greatly reduces mesh-size sensitivities. This also has important ramifications in that accurate results can be achieved at larger cell sizes, which in turn greatly reduces the run-time of simulations.

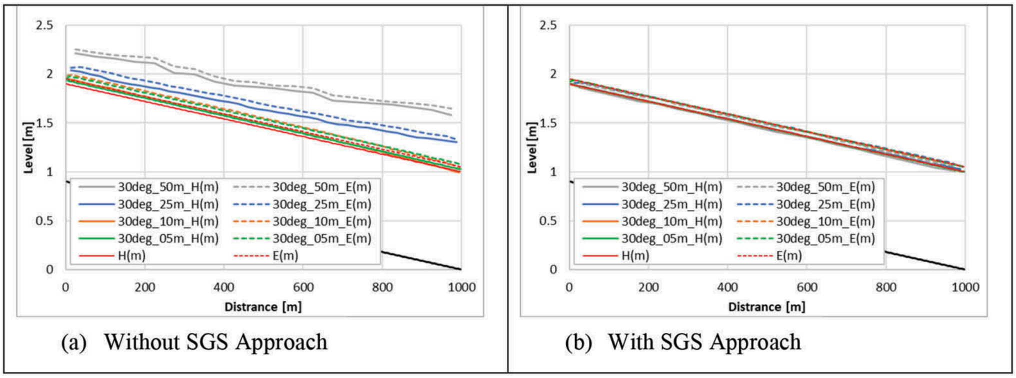

Figure 5. Effect of cell size upon simulated water level in a rectangular channel: (a) without SGS approach and (b) with SGS approach. Charts show energy (dashed lines) and water surface (solid lines).

4.2 *U-bend flume test case*

The U-Bend flume test (De Vriend, 1978) is illustrated in **Error! Reference source not found.**. The channel is 1.7 m wide and has an inside radius of 4.25 m. Water levels at the inside, centre-line and outside were measured every 1 m in longitudinal length or at 15° intervals around the bend. The flow applied was 0.189 m^3/s with a downstream water depth of 0.18 m. A Manning's *n* value of 0.0125 was used, based on the published roughness height of 0.001 to 0.0005 m.

For the non-SGS approach the model results demonstrated the influence of the same jagged saw-tooth effect (shown in Figures 2 and 3) along the curved wet-dry boundary of the bend. As for the rectangular channel case the modelled water levels were higher than those measured due to the distorted velocity field (Figure 7(a) and 7(c)). However, the SGS approach with its ability to represent partially wet cells and cell faces produced a much improved comparison against measured data as shown in Figure 8(c). The impact of the saw-tooth wet-dry boundary around the U-Bend is non-existent as illustrated in Figure 8(b) due to the wet-dry boundary being represented by partially wet cells and flows conveyed through partially wet cell faces resulting in a smooth flow field. Also noticeable is that the SGS approach produces water levels that are much smoother due to the improved velocity field.

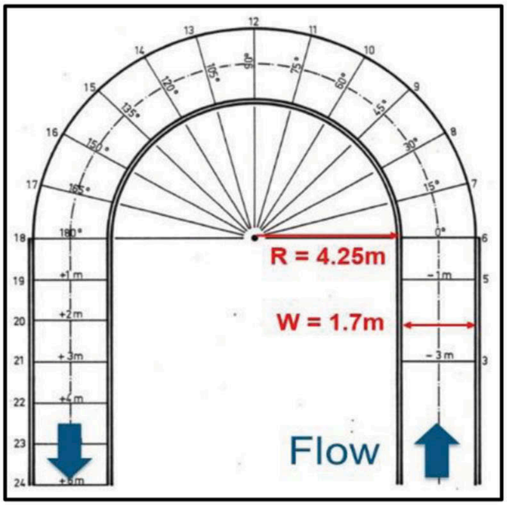

Figure 6. U-bend test configuration.

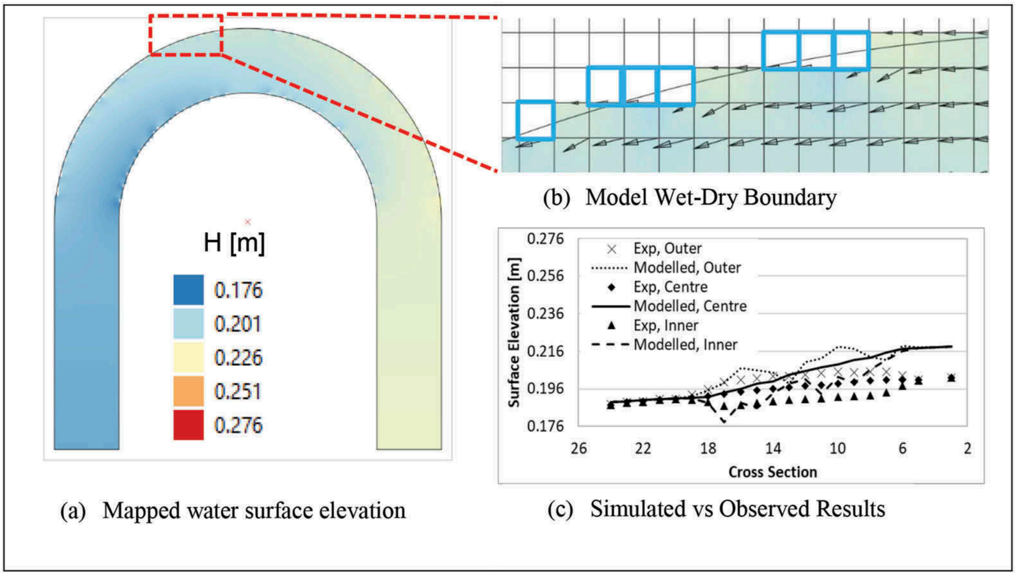

Figure 7. U-shape bend result for the non-SGS scenario. Note the jagged wet-dry boundary in (b) and the higher than observed water surface elevation in (c). The highlighted blue cells have centroids outside the edge of the flume, so they are modelled as permanently dry or inactive using the non-SGS approach.

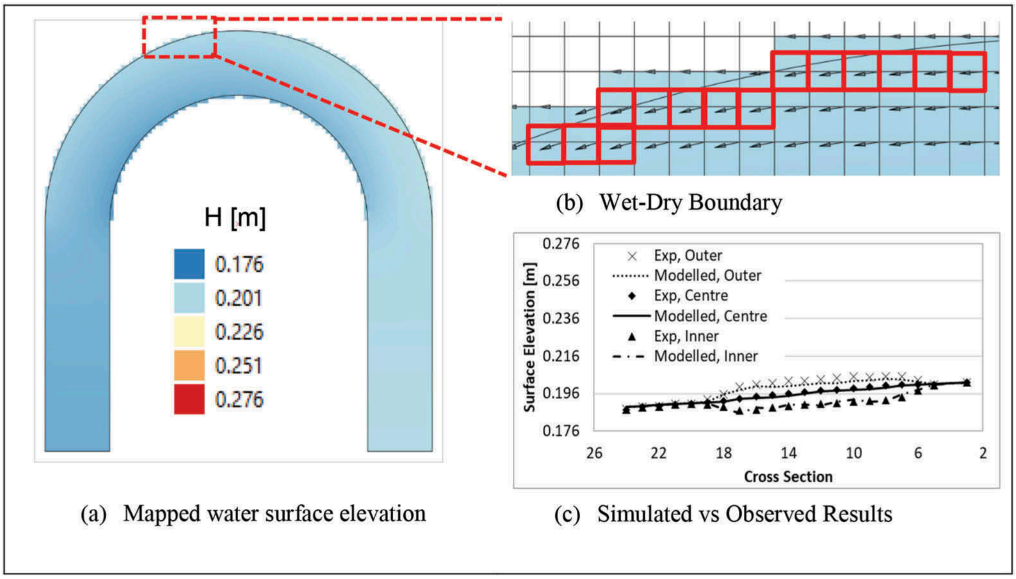

Figure 8. U-shape bend result for the SGS scenario. All the highlighted red cells in (b) have partially wet cells and faces based on the shape of the flume. Note the jagged wet-dry boundary exists but the highlighted cells in Figure 7 (b) are now partially wet and convey water. The simulated water levels are in much greater agreement with the observed water surface elevation (c).

The model was simulated for cell sizes of 0.05, 0.10, 0.17 and 0.34 m to assess dependency on mesh resolution. Using the non-SGS approach, none of the cell sizes reproduced the measurements with all cases over predicting the upstream water level and for the coarser cell sizes the

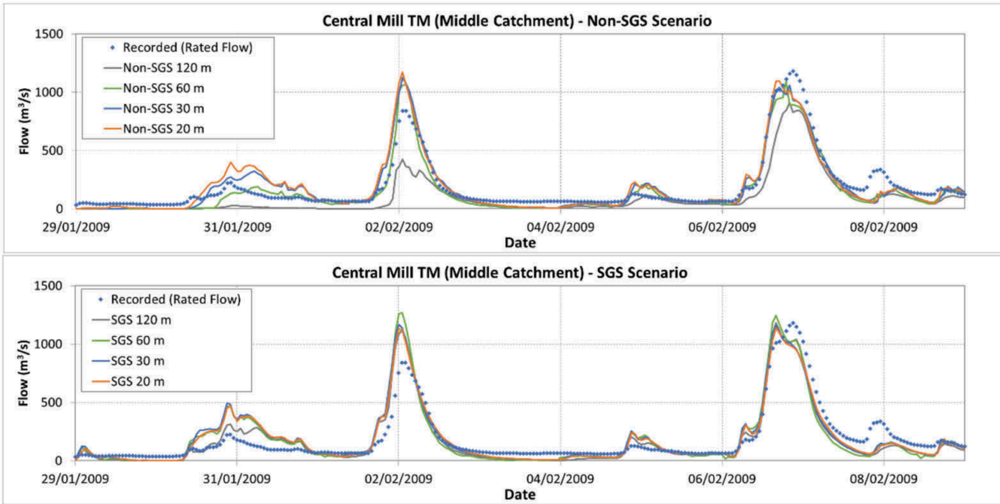

Figure 9. Improvement in catchment response for larger cell sizes using SGS.

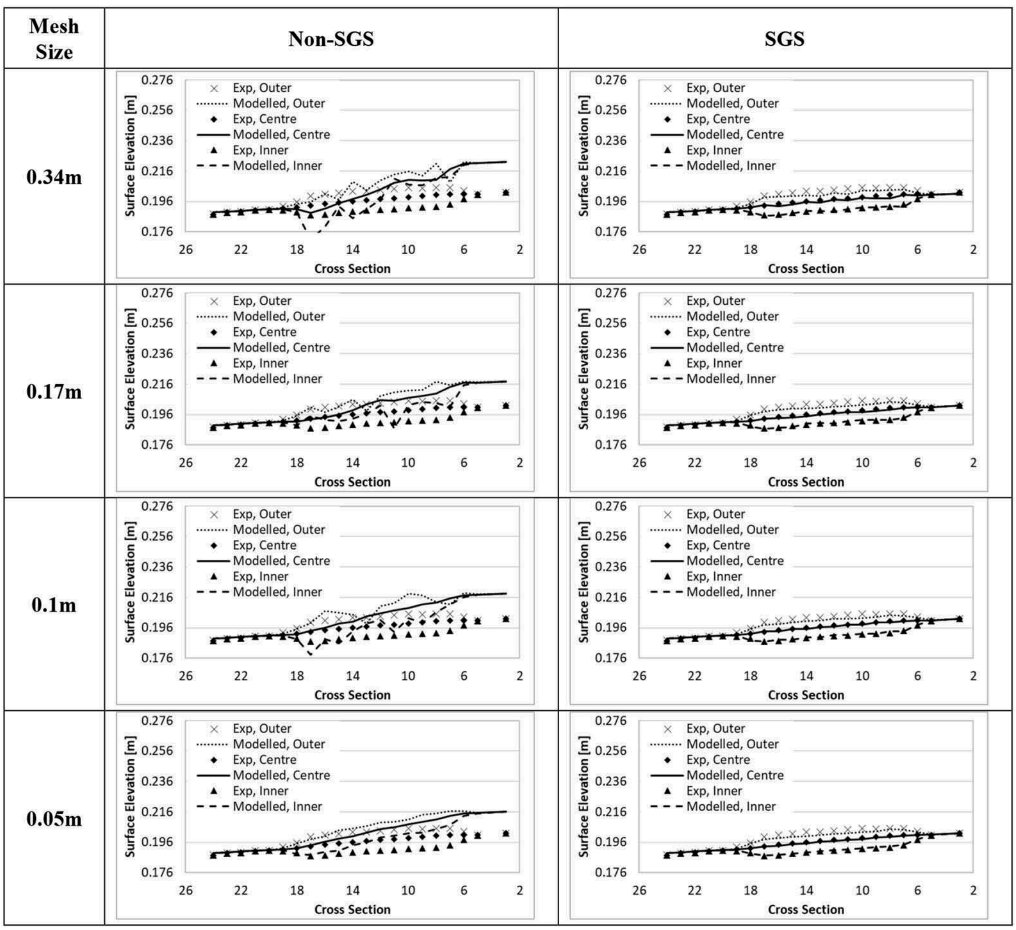

Figure 10. Results for the U-Shape Bend Test Case for a Range of Mesh Size Resolution for both the Non-SGS and SGS cell schematisation.

water surface was significantly disturbed as shown in Figure 10. In contrast, the SGS approach correlated well with the measurements for all cell sizes as shown in the right-side charts.

4.3 *Real-world applications*

Results from real-world applications have thus far indicated superior performance using the SGS approach. Sub-cell flow paths such as a 2 m stream within a 20 m cell or a small urban channel are now seemingly well represented. Whole of catchment direct rainfall models flow more responsively with water not being "trapped" by a too coarse cell resolution, providing a better representation of the low flow rising limb and improved simulation of the flood arrival time at gauging stations (Figure 9).

Primary flow paths such as rivers and open channels can be represented at coarser resolutions and achieve similar results, meaning the calibration of models can now proceed at a coarser resolution than before, enabling faster turnover of test simulations. Simulations using a range of grid sizes have been showing much improved mesh cell size convergence.

5 CONCLUSIONS

Results from the theoretical and flume-scale benchmark tests presented, along with observations from other benchmarking and real-world applications, demonstrate that the addition of a sub-grid scale approach to sampling terrain and bathymetry can provide substantial benefits to regular (cartesian) mesh 2D solvers. The more detailed representation of the cell volume and face flux area/width from the sub-grid sampling (SGS), compared with the traditional single elevation per cell and per face approach, significantly reduces or removes, the sensitivity of regular meshes to mesh orientation and mesh cell size. With SGS a 2D regular mesh model can be rotated or have a change in cell size with acceptable changes in results compared with much greater and sometimes unacceptable changes in results for the single elevation per cell approach.

The effect of SGS on the wet-dry boundary where the flow is at a different orientation to the mesh is to produce a smooth, continuous flow field, with no distorted flows. The results from the benchmarking show that without SGS, results are sensitive to both the mesh resolution and orientation when simulating flows within narrow channels. Testing of the SGS approach suggests that flows within narrow channels are significantly less sensitive to both mesh resolution and alignment, obey Manning's formula and more closely match experimental measurements.

Results from real-world applications suggest that the SGS approach provides a better definition of low/shallow flows and therefore provides a more accurate representation of the response times to a rainfall event, with little water retention in the upper catchments, which can occur using non-SGS methods. The SGS approach appears to improve the representation of flow mechanisms within the model domain avoiding disconnected cells that retain water and unrealistic velocity values along the wet-dry boundary. The cell size independency has greatly assisted in the calibration of real-world models to observed data, as calibration simulations can initially proceed at much faster compute times using a coarser resolution than conventionally used.

The findings indicate regular mesh solvers can produce the same quality as a well-designed, high resolution, irregular mesh whilst maintaining the benefits of a regular mesh in terms of far greater efficiency of model setup and options analysis. The use of the SGS approach also appears to have benefits in the requirement of fewer cells, faster simulations whilst maintaining accurate modelling of fine-scale features. The representation of high-resolution topographic data with SGS whilst maintaining a practical grid cell size together with the elimination of mesh size and orientation sensitivity will improve the accuracy of flood hazard prediction without the need for increasing mesh resolution or mesh sensitivity analysis, both of which require time consuming efforts either computationally or with processing. Modelling at larger mesh sizes and the associated faster simulations open up the potential for more accurate real-time simulation as well as ensemble runs for the purpose of uncertainty analysis. Whether utilised independently or in conjunction, the SGS benefits would improve the prediction and understanding of floods hazard at a range of scales and aid flood risk assessment, management and mitigation in the face of increased flood risk.

REFERENCES

Alfieri, L., Dottori, F., Betts, R., Salamon, P., and Feyen, L., (2018). Multi-Model Projections of River Flood Risk in Europe under Global Warming, Climate 6(1).

Australian Rainfall and Runoff (2012), Project 15, Two Dimensional Modelling in Urban and Rural Floodplains, Stage 1&2 Report, P15/S1/009, November 2012.

Bates, P., Horritt, M., and Hervouet, J., (1998), Investigating two-dimensional, finite element predictions of floodplain inundation using fractal generated topography, Hydrological Processes, 12 (8),1257–1277.

Collecutt, G., and Syme, W.J., (2017). Experimental benchmarking of mesh size and time-step convergence for a 1st and 2nd order SWE finite volume scheme. Proceedings of the 37th IAHR World Congress, KL.

Collecutt, G., and Syme, W.J., (2020). Turbulence Modelling for the 2D Shallow Water Equations. Submitted paper for the IAHR 10th Conference on Fluvial Hydraulics, River Flow 2020, Delft, Netherlands.

Centre for Research on the Epidemiology of Disaster (CRED), (2019), Natural Disasters 2018. Brussels.

De Vriend, H.J. and Koch, F.G., 1978. Flow of water in a curved open channel. Delft Hydraulics Laboratory/Delft University of Technology, TOW-Report, R657, VII/M141S-III.

Lane, S., (1998), Hydraulic modelling in hydrology and geomorphology: A review of high-resolution approaches, Hydrological Processes, 12(8),1131–1150.