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Linked Two-Dimensional/One-Dimensional Flow Modelling Using the Shallow Water Equations

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SUMMARY

The ability to represent a hydrodynamic system as a combination of 1-D and 2-D models run in unison has numerous benefits relating to cost and accuracy. To address this issue a computer program, codenamed TUFLOW, for simulating depth averaged, 1-D and 2-D unsteady free-surface flows has been developed. Areas of complex flows are computed over a 2-D rectangular grid which may have attached to it any number of 1-D network models. The 2-D solution technique is based on the scheme by Stelling (1984) while coding from the hydrodynamic program ESTRY was utilised for the 1-D network component.

Details of the TUFLOW program development and testing including the 1-D/2-D interface algorithm are presented. Results of the work to date indicate the Stelling scheme to be a major improvement on the well-known RAND Corporation program (Leendertse, 1967) with respect to stability, robustness and boundary representation. Testing and practical application has shown the 1-D/2-D interface algorithm to be accurate, versatile and a powerful feature for modelling complex flow patterns in estuaries and rivers.

1. INTRODUCTION

Significant benefits would be realised when undertaking numerical studies of complex flow patterns in estuaries and rivers if the hydraulic system can be represented by a 2-D model linked to one or more 1-D models. The area of complex flow can thus be modelled using a 2-D solution algorithm with the remainder of the waterway represented by the more economical 1-D solution.

A joint research project between WBM Pty Ltd (Oceanics Australia) and The University of Queensland Civil Engineering Department was initiated in April, 1989 to develop a 1-D/2-D hydrodynamic modelling package. This paper presents a brief discussion of the work completed by December, 1989 as summarised below.

(a) Research and development of computer software for computing depth averaged, 2-D flows.

(b) Development of a comprehensive computer graphics pre- and post-data processor so as to minimise data input and enhance the presentation of results for 2-D models.

(c) Research and development of a methodology to interface the 2-D solution scheme with that of a 1-D hydrodynamic network program.

2. 1-D/2-D INTERFACE CONSIDERATIONS

Prior to development of computer software a number of issues such as timestep compatibility and the suitability of the various 1-D and 2-D solution schemes available were addressed. These issues and the approach adopted are presented as follows:

(a) It is necessary for the 1-D and 2-D solutions to be time-step compatible to avoid extrapolation of boundary values at the interface. This would generally necessitate the 1-D solution to be run at a lower Courant number than the 2-D solution because of the smaller spatial discretisation of 2-D models. The Courant number (Cr), which is defined below, is used as a guide for selecting computational time steps in hydrodynamic modelling. Explicit programs are restricted to Courant numbers less than unity. Implicit schemes are in theory not restricted by the Courant condition but some schemes become inaccurate at high Courant numbers.

$Cr = \Delta t\sqrt{(gH)}/\Delta x$ for a 1-D solution
$Cr = \Delta t\sqrt{(2gH)}/\Delta x$ for a 2-D solution over a square grid

where Δt = time step (s)
g = acceleration due gravity (m^2/s)
H = depth of water (m)
Δx = length of grid (m)

(b) Point 1 above implies that it would be desirable for the 2-D solution scheme to be implicit allowing the use of large timesteps. For the 1-D scheme an implicit scheme would also be desirable giving greater flexibility when selecting timesteps. However, the Courant number for the 1-D model may commonly be less than unity allowing the use of an explicit scheme. The advantage of an explicit scheme is that it requires less computational effort per time step than an implicit scheme.

(c) On the basis of the above and the availability of the source code, the hydrodynamic network program ESTRY which uses an explicit finite difference solution of the 1-D hydrodynamic equations was chosen. ESTRY is an established and proven program for modelling of tides and floods in estuaries and rivers.

(d) For the 2-D solution scheme, use of the RAND corporation program (Leendertse, 1967) was considered initially since it was available. A number of inadequacies of this scheme which are quoted in the literature and have been experienced by the authors and others suggested it would be beneficial to investigate the development or purchase of software utilising a more advanced solution technique.

(e) After reviewing the literature and investigating the availability of recently developed programs it was decided to develop a new 2-D hydrodynamic program, codenamed TUFLOW, using the solution scheme of Stelling (1984).

3. ESTRY

ESTRY is a proven and versatile program for modelling the hydrodynamic processes of floods and tides. It has undergone extensive testing and has been successfully applied to a large range of investigations along the eastern coast of Australia.

ESTRY represents an area as a combination of nodes and channels where the nodes model the storage characteristics and the channels the flowpaths. Channels and nodes can be linked to represent floodplains and river confluences in a quasi 2-D manner. Algorithms for simulation of flow through bridges, culverts and weirs and for free-overfall across natural levee banks are available.

An explicit Runge-Kutta solution technique is employed to solve the 1-D equations of continuity and momentum (Morrison, 1978) as presented below.

$$\frac{\partial (uA)}{\partial x} + B\frac{\partial \zeta}{\partial t} = 0 \qquad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + g\frac{\partial \zeta}{\partial x} + k|u|u = 0 \qquad (2)$$

where ζ = water surface level
u = depth and width averaged velocity
A = cross-sectional area of channel
B = width of channel
k = friction coefficient (based on Manning's n and a dynamic head loss)
x = distance along channel
t = time

4. TUFLOW

The development of the program TUFLOW for analysing depth averaged, two-dimensional unsteady flow was based on the work of Stelling (1984). Stelling uses an Alternating Direction Implicit (ADI) method to solve the equations of momentum and continuity in two-dimensions. The scheme is similar to the well known method of Leendertse (1967) but uses a higher order of accuracy and a more comprehensive treatment of the boundaries giving a significantly more robust and versatile solution.

TUFLOW solves the 2-D shallow water equations of continuity and momentum which are of the following form.

$$\frac{\partial \zeta}{\partial t} + \frac{\partial (Hu)}{\partial x} + \frac{\partial (Hv)}{\partial y} = 0 \qquad (3)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv + g\frac{\partial \zeta}{\partial x} + gu\frac{\sqrt{(u^2 + v^2)}}{C^2 H} - \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) = F_x \qquad (4)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu + g\frac{\partial \zeta}{\partial y} + gv\frac{\sqrt{(u^2 + v^2)}}{C^2 H} - \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) = F_y \qquad (5)$$

where ζ = water surface level
u = depth averaged velocity in x-direction
v = depth averaged velocity in y-direction
h = depth of water relative to a datum
$H = \zeta + h$ = total water depth
f = coriolis parameter
C = Chezy bed friction coefficient
F = External Force = $F_w + F_r + F_p$
F_w = External wind force
F_r = Force due to waves (radiation stress)
F_p = Force due to barometric pressure

The reader is referred to the work of Stelling (1984) for full details of the solution scheme.

4.1. Testing of TUFLOW

The TUFLOW program coding has been verified by testing under idealised cases and by comparison with cases presented in the literature. The idealised cases have confirmed the accuracy of the scheme with respect to the friction, advection and propagation terms. Testing to cases presented by Stelling (1984), Benque (1982) and Weare (1979) have also confirmed the correct representation of these terms, including diffusion, along with the known limitations of ADI schemes.

The results from the TUFLOW program of a test case presented in Stelling (1984) is shown in Figure 1. The test case illustrates the formation of eddys behind a solid wall.

A wetting and drying algorithm, similar to that of type II described by Stelling (1986), has also been implemented.

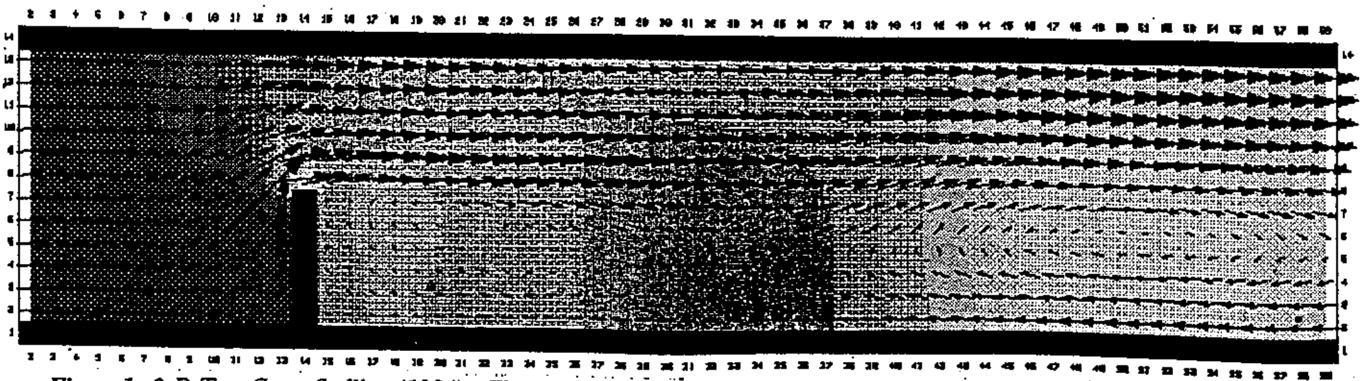

Figure 1. 2-D Test Case, Stelling (1984) - Flow past a Solid Wall - Velocity Vectors superimposed on Water Level Contours

An example of the practical application of TUFLOW is presented in Figure 2, which shows the ebb tide velocities in Moreton Bay, Qld. The black and white shading represents high ground, dry intertidal flats and water depth. The model is composed of 133 by 89 square grids at a spacing of 500m.

5. COMPUTER GRAPHICS SYSTEM

To increase the efficiency of the research and development programme a comprehensive menu-driven computer graphics system was developed for the 2-D models. For the 1-D network program, ESTRY, a detailed I/O system had already been developed (Syme, 1989). The major features of the graphics system for 2-D models are summarised below.

(a) Data input of bathymetry, etc. is carried out by digitising spot values and contours. Values at each grid location are calculated by interpolation between these contour lines and the spot values or entered by direct input. Large models may be digitised in sections using the various zoom functions.

(b) Presentation of input data or the results of one or more simulations in the form of water levels, velocities and actual water depth can be illustrated as any combination of the following formats:
- numerical values
- vector plots
- line contours
- colour shaded contours
- 3-D colour shaded perspectives

(c) General features include:
- zoom functions which are essential for large models.
- windowing for easy comparison of results of different simulations, or at different times of the one simulation
- particle tracking

6. 1-D/2-D INTERFACE

The linking of 1-D and 2-D models to run in unison is an area of hydrodynamic computer modelling which has been addressed in relatively only minor detail to date.

In studies where both 1-D and 2-D models have been used, the boundary conditions for the 2-D model at the 1-D/2-D interface have been determined from the results of a separate 1-D modelling exercise. This is generally a satisfactory arrangement provided that, when development proposals are assessed, impacts do not propagate from one model to the other.

The advantages of being able to run 1-D and 2-D models in unison are;-

(a) The coverage or extent of the 2-D model will generally be less, reducing computation time and/or allowing smaller grid sizes leading to greater accuracy. The smaller coverage is due to the greater flexibility in choosing suitable boundary locations.

(b) Hydraulic impacts due to development proposals will be represented throughout both 1-D and 2-D models allowing a more comprehensive and accurate assessment.

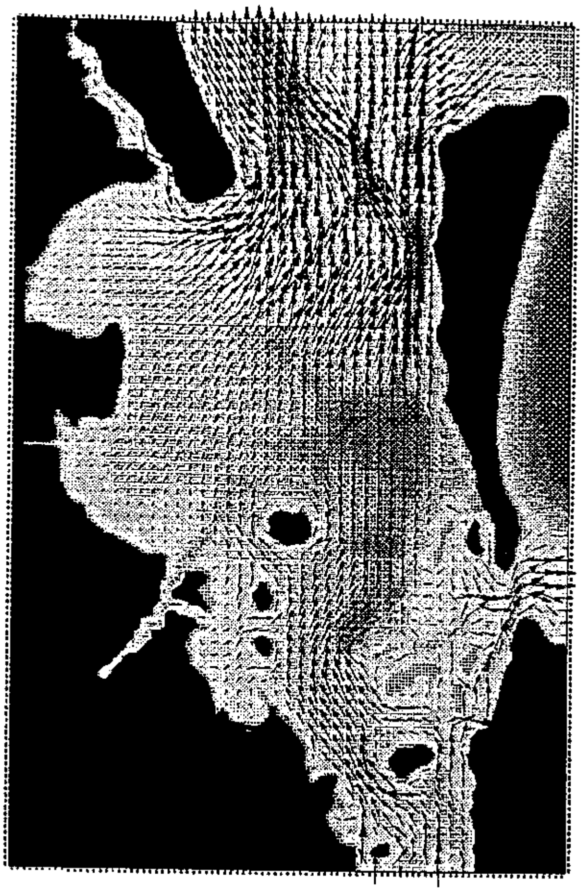

Figure 2. Moreton Bay Currents during an Ebb Tide Velocity Vectors superimposed on the Bathymetry

(c) The linkage of 1-D and 2-D models is a more versatile and flexible system enhancing user satisfaction and performance.

6.1. 1-D/2-D Interface Algorithm

Hydrodynamic models are driven at their boundaries by either a water level or flow. Flow boundaries can be described by velocities or discharges.

At a 1-D/2-D interface the boundary type of the 1-D and 2-D models must be defined. The options available are intuitively a 1-D level/2-D level, 1-D flow/2-D flow, 1-D level/2-D flow or 1-D flow/2-D level.

The adoption of either a 1-D level/2-D level or a 1-D flow/2-D flow boundary arrangement can immediately be ruled out as either the flow or water level at the interface will remain undefined throughout the computation period.

This leaves either a 1-D level/2-D flow or 1-D flow/2-D level arrangement. The latter configuration is easier to implement as no knowledge of the velocity distribution across the 2-D model boundary is required. It is also noted that in reality, little or no information about the velocity distribution would be known indicating that it would be difficult to implement a 1-D level/2-D flow interface.

The selection of the 1-D flow/2-D level interface was further supported by the testing of the 2-D program which showed a water level boundary produced more stable and accurate results than a flow boundary.

Based on the above, the 1-D flow/2-D level arrangement at the interface was chosen. The arrangement functions by conveying to the 1-D model at the interface the net flow across the boundary of the 2-D model. This is followed by the 1-D model specifying a new water level to the 2-D model based on the 1-D hydrodynamic computations. The 2-D model is resimulated using the new level and the process above is repeated.

Of particular note is that the water level calculated by the 1-D solution at a node represents a static water level while the 2-D scheme closely approximates the true water surface level ie. static level less dynamic head. To allow for this at the 1-D/2-D interface, the water level specified for the 2-D boundary by the 1-D model is reduced by the dynamic head ($u^2/(2g)$).

The solution techniques of both ESTRY and TUFLOW are congenial for interfacing as they both utilise a two half time-step solution methodology allowing the procedure described above to be carried out for each half time step.

The TUFLOW program coding incorporates the necessary coding from ESTRY, and has been developed such that any number of ESTRY 1-D network models may be attached to a TUFLOW 2-D model.

6.2. Testing 1-D/2-D Interface

Testing of the 1-D/2-D interface at the time of writing this paper has been carried out to several idealised cases and practical situations.

The results of testing using idealised models have shown the 1-D/2-D interface to have no adverse effect on the hydrodynamics of a system. The approach has generally been to set up a 1-D network model and quantify the hydraulics in terms of water levels, velocities, flows and the integral of flow versus time. Part of the 1-D model is then replaced by a 2-D model and the hydrodynamics are re-simulated. Comparison of the results between the two simulations have consistently shown the 1-D/2-D interface to be performing correctly.

Testing has also shown that a 1-D/2-D interface can be specified at an angle to the X or Y axes of a 2-D grid and still maintain a correct representation of flow across the interface. This is an important aspect in practice as 'unfortunately' water courses do not lie at right angles.

An example of a system being represented by a combination of 1-D and 2-D models is shown in Figure 3. The idealised tidal system represents a basin which is connected to the ocean (Node 1) by Channels 1 and 2. The other channels lead to two storages represented by Nodes 13 and 23. Two Cases A and B are considered to establish the impact of increasing the tidal prism of the system. To achieve this the surface area of Node 23 has been increased from 20ha in Case A to 500ha in Case B. The system is driven by a sinusoidal ocean tide of amplitude 2m and period 12.5h.

The results of Cases A and B are presented in Figures 4 and 5. The tidal range of the system is significantly changed because of the increase in tidal prism as shown for node 13 in Figure 4. It can also be seen that a change in flow patterns through the basin occurs as illustrated by Figures 5a and 5b.

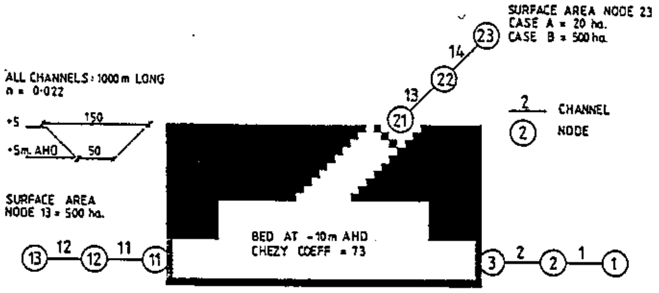

Figure 3. Idealised 1-D/2-D Model Configuration

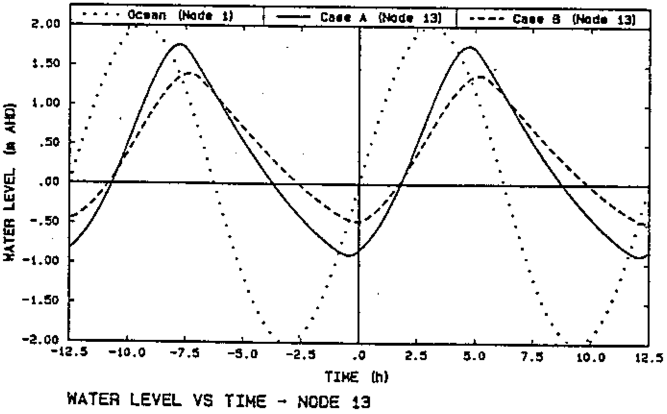

Figure 4. Impact on Tidal Range at Node 13

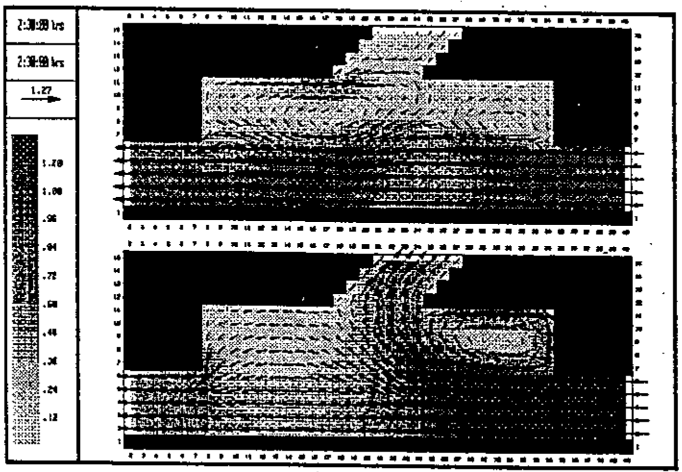

Figure 5a. Flow Patterns during Flood Tide
Case A above, Case B below
Velocity Vectors and Contours

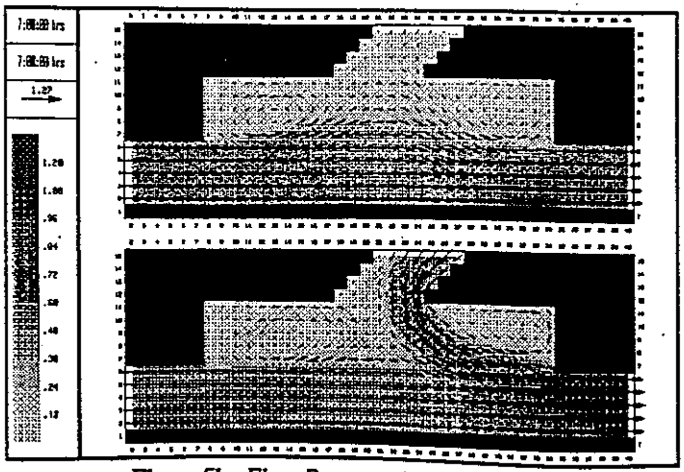

Figure 5b. Flow Patterns during Ebb Tide
Case A above, Case B below
Velocity Vectors and Contours

A practical application of the 1-D/2-D linkage has been carried out for the entrance of the Tweed River, NSW. An ESTRY 1-D network model was developed, calibrated and verified for the Department of Public Works, NSW (4). As a test for the 1-D/2-D interface, a 2-D model of the entrance was developed and linked to the 1-D network model as shown in Figure 6. The 1-D model consists of 94 channels and 80 nodes while the 2-D model is represented by a 66 by 43 grid with a spacing of 30m.

The 1-D/2-D system was run for a sinusoidal mean spring ocean tide for three tidal cycles to show equilibrium was reached. After fine tuning the bed friction, good comparison of results between the calibrated ESTRY model and the combined 1-D/2-D model was reached. It is interesting to note that to calibrate the ESTRY model separate channels to represent the flood and ebb tide flows were required to model the 'jet' effect of the training walls. This was not required for the 1-D/2-D system as the 2-D model takes into account such an effect as illustrated in Figure 7.

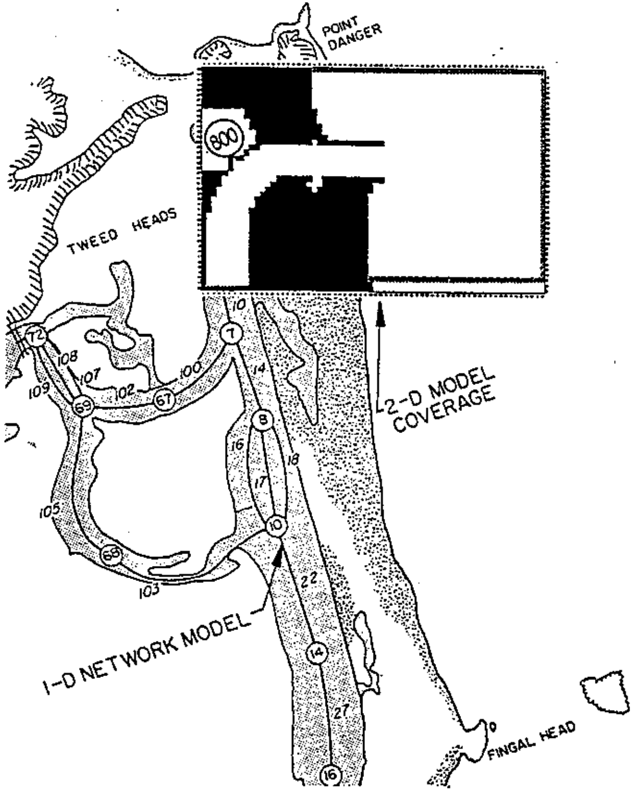

Figure 6. 1-D/2-D Model Interface for Tweed Entrance Case

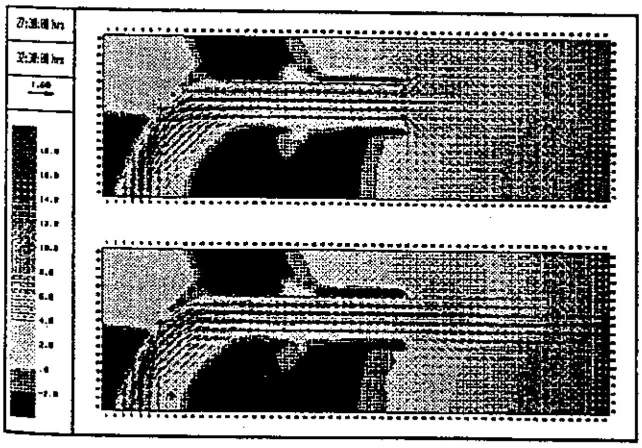

Figure 7. Flood & Ebb Tide Current Patterns - Tweed Entrance Velocity Vectors superimposed on the Bathymetry

7. CONCLUSIONS

A computer program, code named TUFLOW, has been developed for analysing depth averaged two-dimensional unsteady flows over a regular grid. The solution scheme is based on the work by Stelling (1984). The results of testing and in its practical application has shown the scheme to be significantly more robust, versatile and accurate than that employed in the well known RAND Corporation program (Leendertse, 1967).

The linkage of the 1-D finite difference solution scheme used in the program ESTRY with the TUFLOW scheme has been successfully implemented and shown to produce accurate and reliable results. The TUFLOW coding allows any number of 1-D network models to be linked to a 2-D model. At the 1-D/2-D interface, a flow boundary for the 1-D model and a water level boundary for the 2-D model is used. Testing has shown conservation of mass to be maintained across the interface for boundaries aligned parallel or obliquely to the X and Y axes.

The ability to run 1-D and 2-D models in unison increases markedly the power and capacity of computer modelling to represent hydrodynamic processes of rivers and estuaries accurately. An area of complex flow can be represented by a 2-D model while the remaining areas can be represented by the more economical 1-D models. When assessing development proposals, the impact is registered over the entire system of 1-D and 2-D models.

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