Turbulence modelling for the Shallow Water Equations

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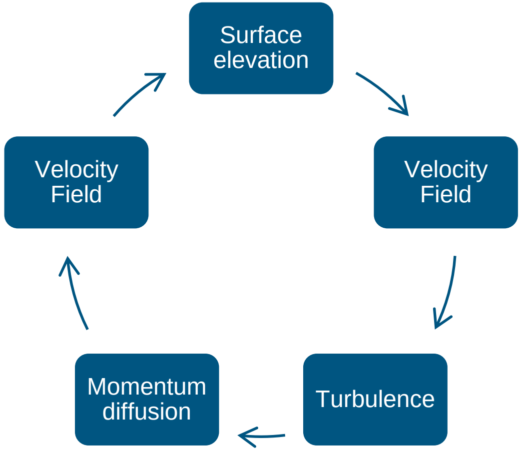
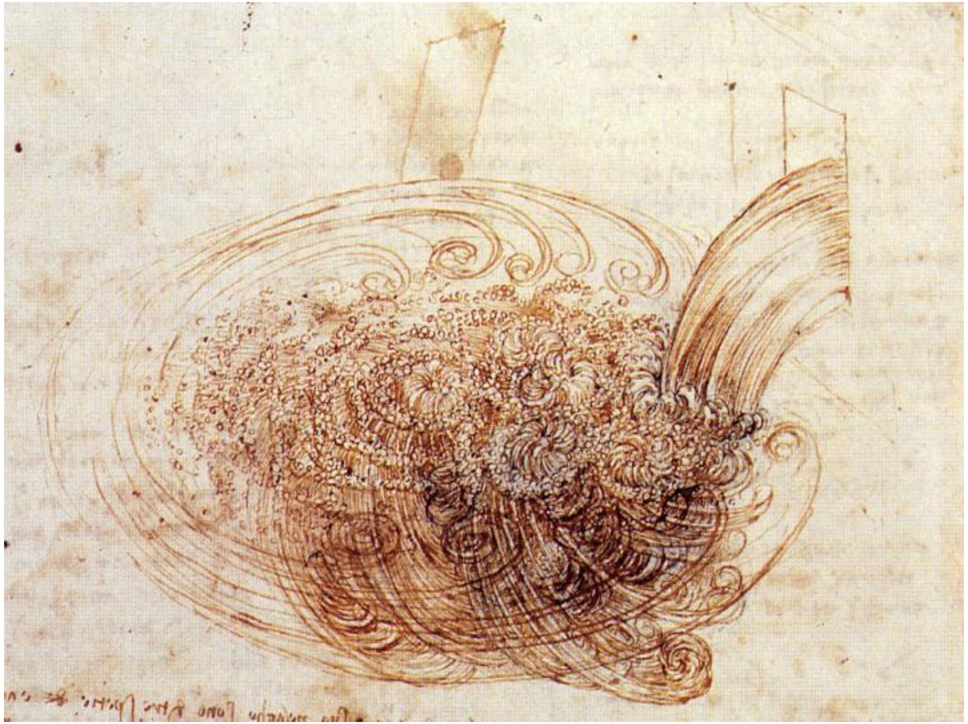
Why Turbulence?

da Vinci, Heisenberg and Lamb all thought turbulence important!

Turbulence causes:

- **almost infinite flow detail**
- **momentum diffusivity**





Current trends in hydraulic modelling

Very coarse mesh

Possibly low-order spatial interpolation

Eddy viscosity model has low impact on results

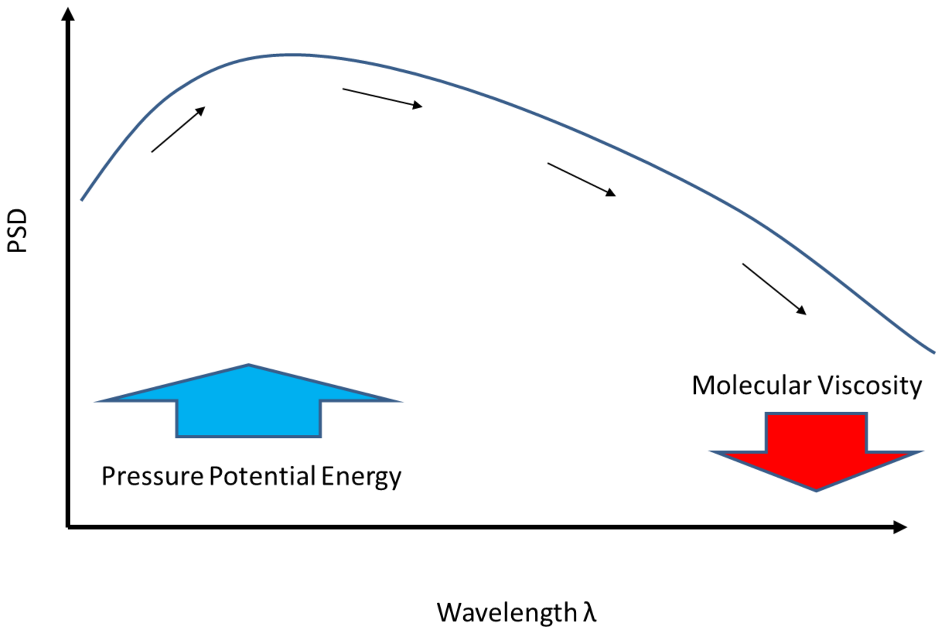
Mesh resolutions commonly less than water depth

Higher-order spatial interpolation

Eddy viscosity model has high impact on results

Scale of Turbulence

- Discretised models of fluids required a turbulence closure model
- Boussinesq proposed to replace Reynolds stresses with turbulent eddy viscosity, ν_t
- Prandtl proposed a length scale, l_m
- Length scale evolves
- In unconfined 3D turbulence exhibits “energy cascade”
- Two common modelling approaches: RAS and LES

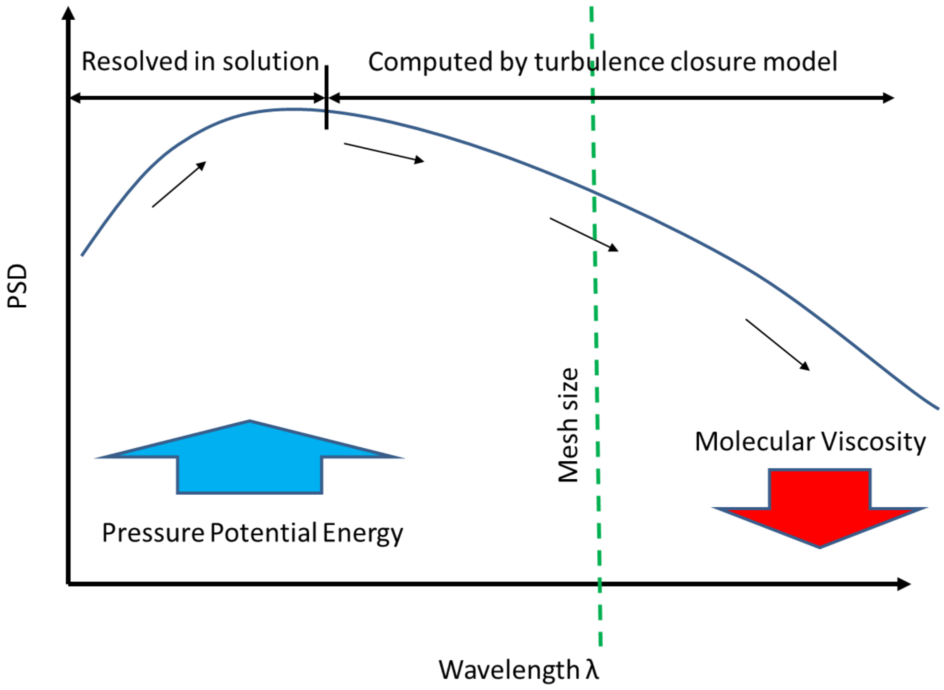


TUFLOW BMT

Reynolds Average Stress (RAS) Turbulence Closure

Solutions spatially smooth

Excellent for steady state solutions or slowly varying in time

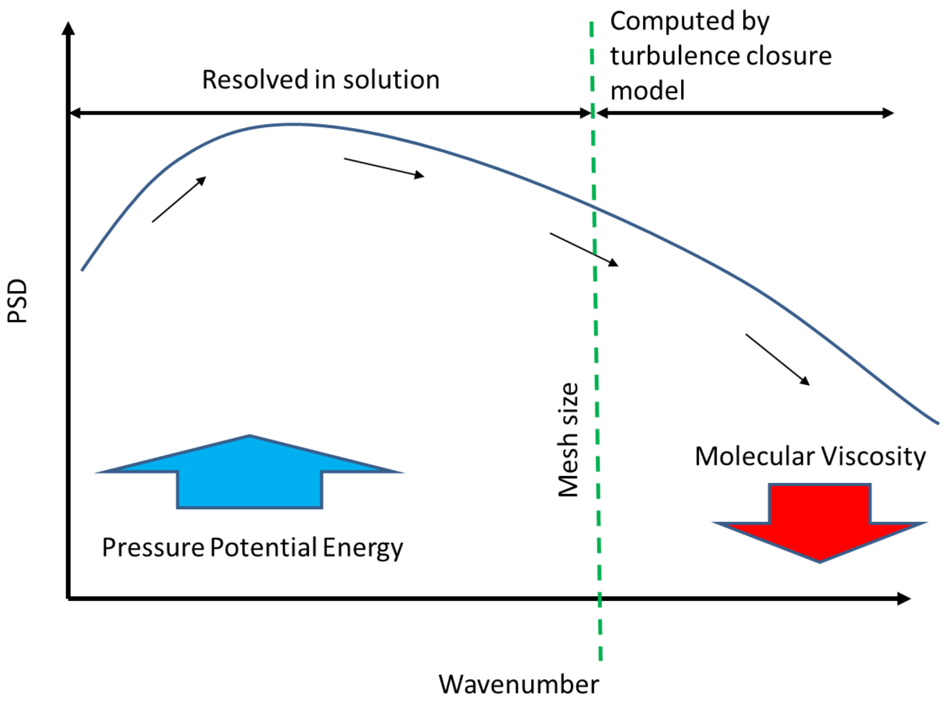


Large Eddy Simulation (LES) Turbulence Closure

Solutions spatially detailed

Excellent for transient solutions

Statistical analysis of transient results sometimes needed



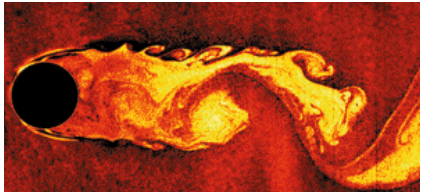
Turbulence in shallow fluid flows

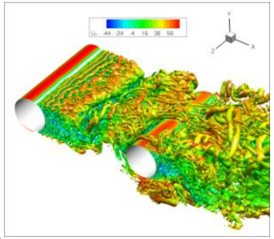
In shallow fluid flows we have both 2D and 3D flow behaviour

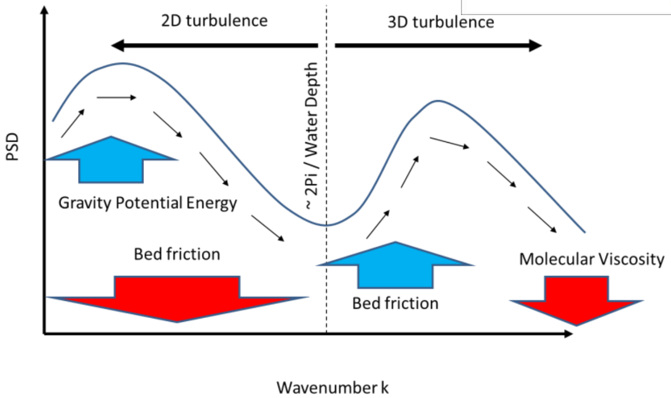
Energy cascade bi-modal

Bed friction converts larger scale 2D turbulence into smaller scale 3D turbulence

Possible minimum in PSD at scales similar to depth







Nadaoka, K., and Yagi, H. (1998). Shallow Water Turbulence Modelling and Horizontal Largey Eddy Computation of River Flow. *J. Hydraulic Engineering*, pages 493–500.

I am about to present …

Three turbulence models

Three benchmark test cases (range of physical scales)

Determine optimum turbulence model parameters for each test case

Summary – is there a ‘one size fits all’ turbulence model?

Constant Viscosity Model

$$\nu_t = \boldsymbol{C}$$

Smagorinsky Turbulence Model

$$v_t = \boldsymbol{M} \Delta x \Delta y |S_{2D}|$$

$$|S_{2D}| = \sqrt{\left(\frac{du}{dx}\right)^2 + \left(\frac{dv}{dy}\right)^2 + \frac{1}{2}\left(\frac{dv}{dx} + \frac{du}{dy}\right)^2}$$

Model is “diagnostic”

Wu Turbulence Model

$$\nu_t = \sqrt{{\nu_{2D}}^2 + {\nu_{3D}}^2}$$

$$\nu_{2D} = \boldsymbol{M_{2D}} {l_m}^2 |S_{2D}|$$

$$\nu_{3D} = \boldsymbol{M_{3D}} l_m U^*$$

$$U^* = \sqrt{\frac{\tau_{bed}}{\rho}} = \frac{|U| n \sqrt{g}}{h^{\frac{1}{6}}}$$

Model is “diagnostic”

$$l_m = min(h, y_{bank})$$

Prandtl Turbulence Model

$$\nu_t = \boldsymbol{M} l_m \sqrt{k}$$

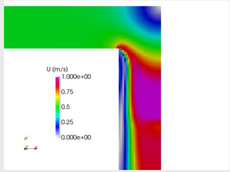
$$\frac{\partial hk}{\partial t} + \underbrace{u\frac{\partial hk}{\partial x} + v\frac{\partial hk}{\partial y}}_{\textbf{Advection}} = \underbrace{\frac{\partial}{\partial x}\left(\nu_t \frac{\partial hk}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu_t \frac{\partial hk}{\partial y}\right)}_{\textbf{Diffusion}} + \underbrace{h\left(P_k - C_D \frac{k^{\frac{3}{2}}}{M l_m}\right)}_{\textbf{Generation/Decay}}$$

$$hP_k = h\nu_t |S_{2D}|^2 + |U|{U^*}^2$$

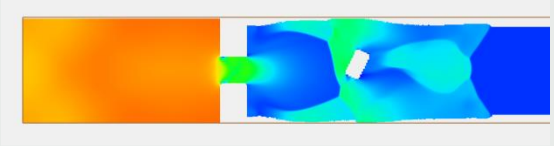
Model is "prognostic" with one additional field "k"

Test Cases

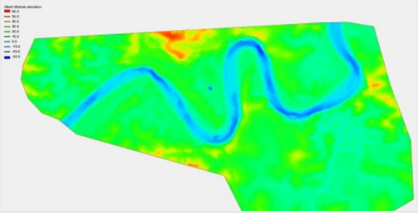
- Right angled flume bend, scale 0.15 m



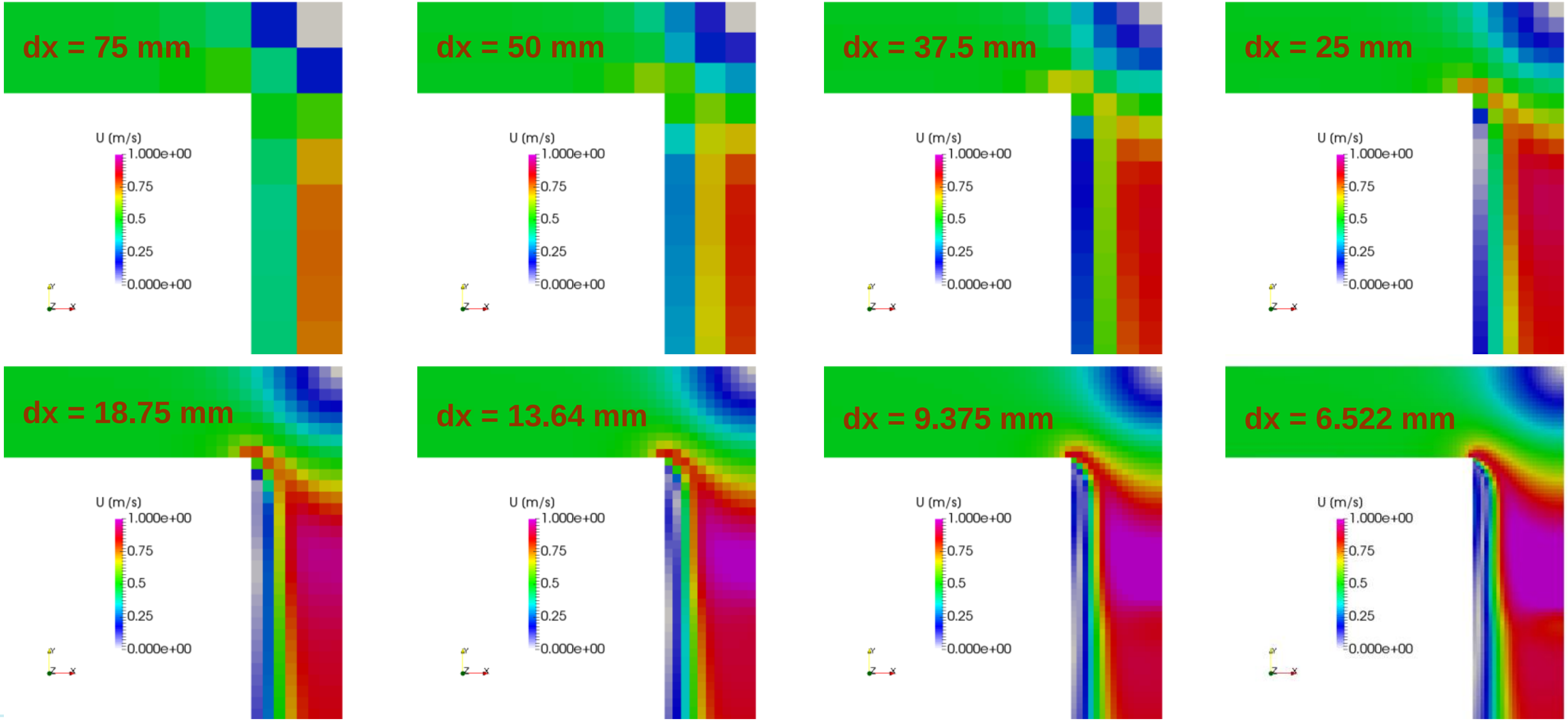
- UK EA T06, scale 3 m



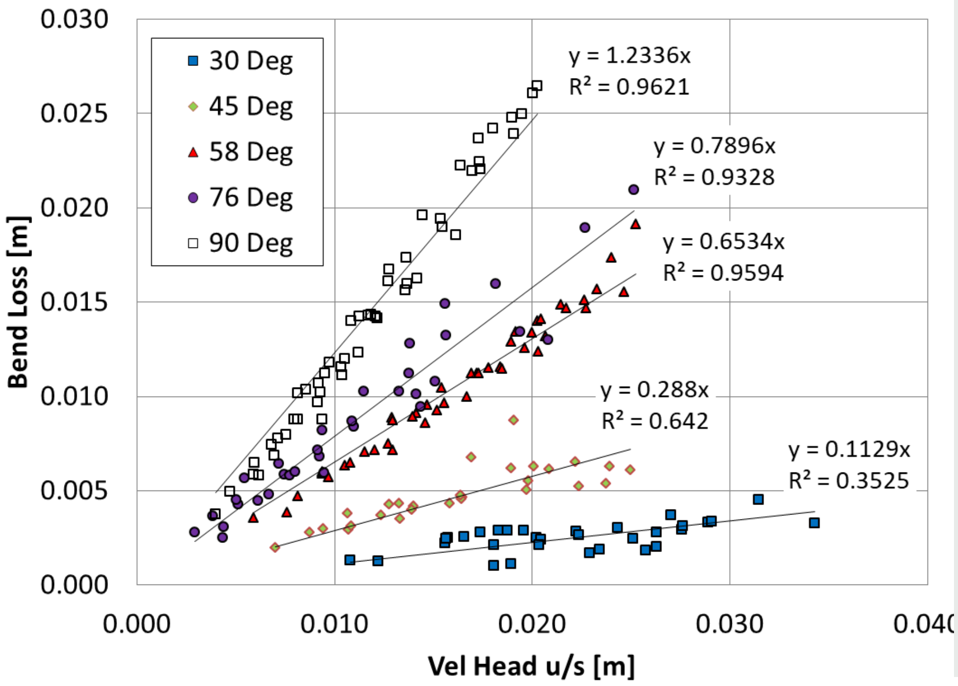
- Brisbane river historical flood event, scale 200 m



Kansas Uni right angled bend flume test
(15 cm wide rectangular section)



Kansas Uni flume test bend results



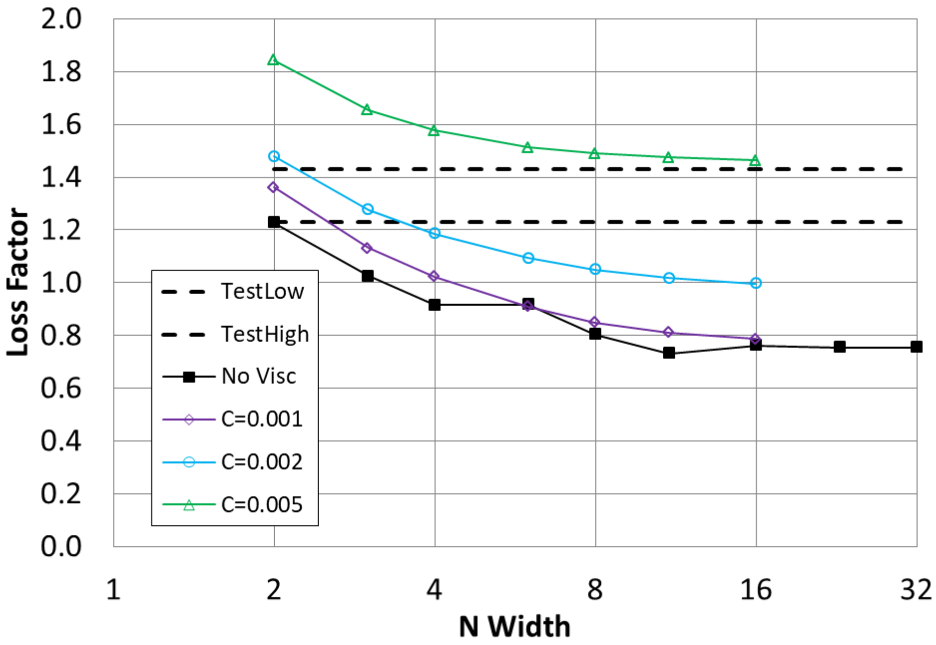
- Malone, T, Parr, D. (2008). Bend Losses in Rectangular Culverts, Kansas Department of Transport (http://ntl.bts.gov/lib/30000/30900/30935/KU-05-5_Final_Report.pdf)
- Excellent correlation between head loss and upstream velocity head
- 90 deg bend loss factor 1.22-1.42

90 Bend Head Loss vs Mesh size

Mesh size converge

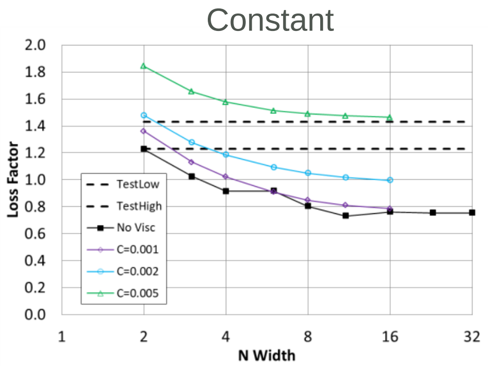
Optimum constant viscosity

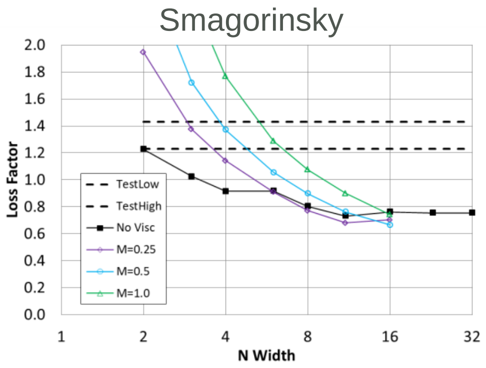
~C=0.004-0.005 m^2/s

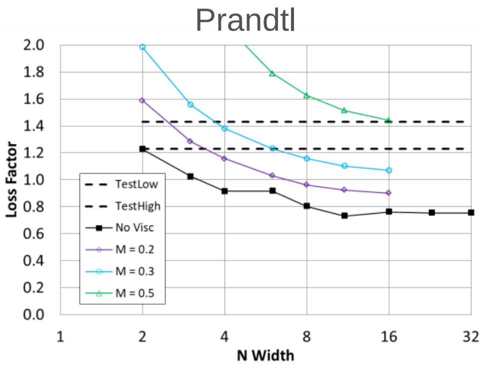


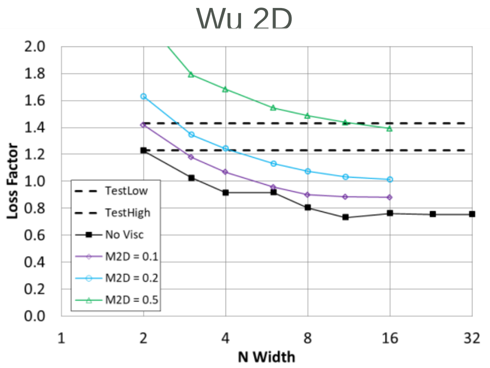
Sesame Street Game

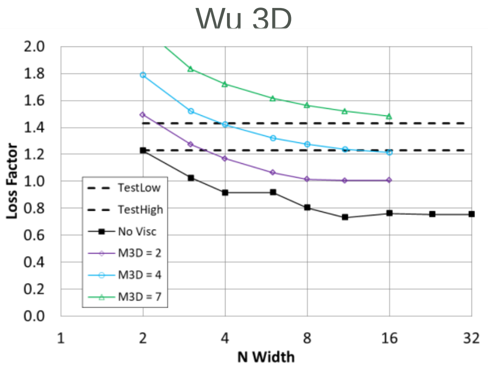
90 Bend Head Loss vs Mesh size











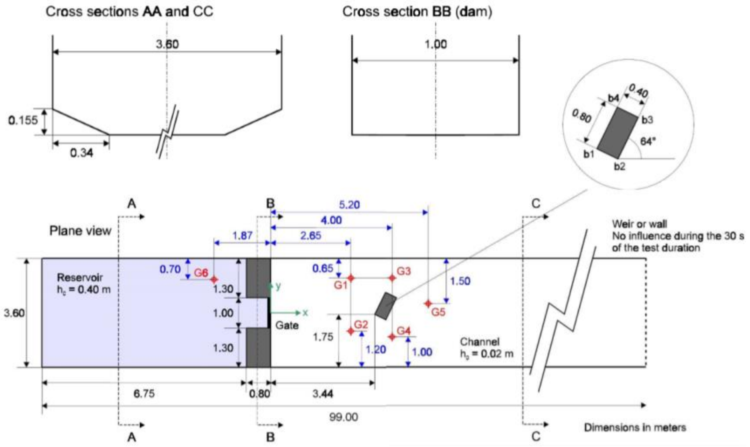
UK EA T06 (Dam Break)

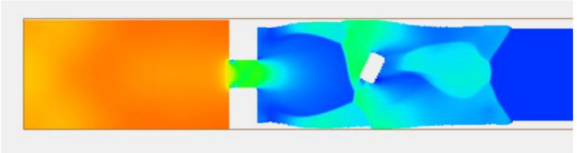
Laboratory scale (~ 3m)

Highly transient event

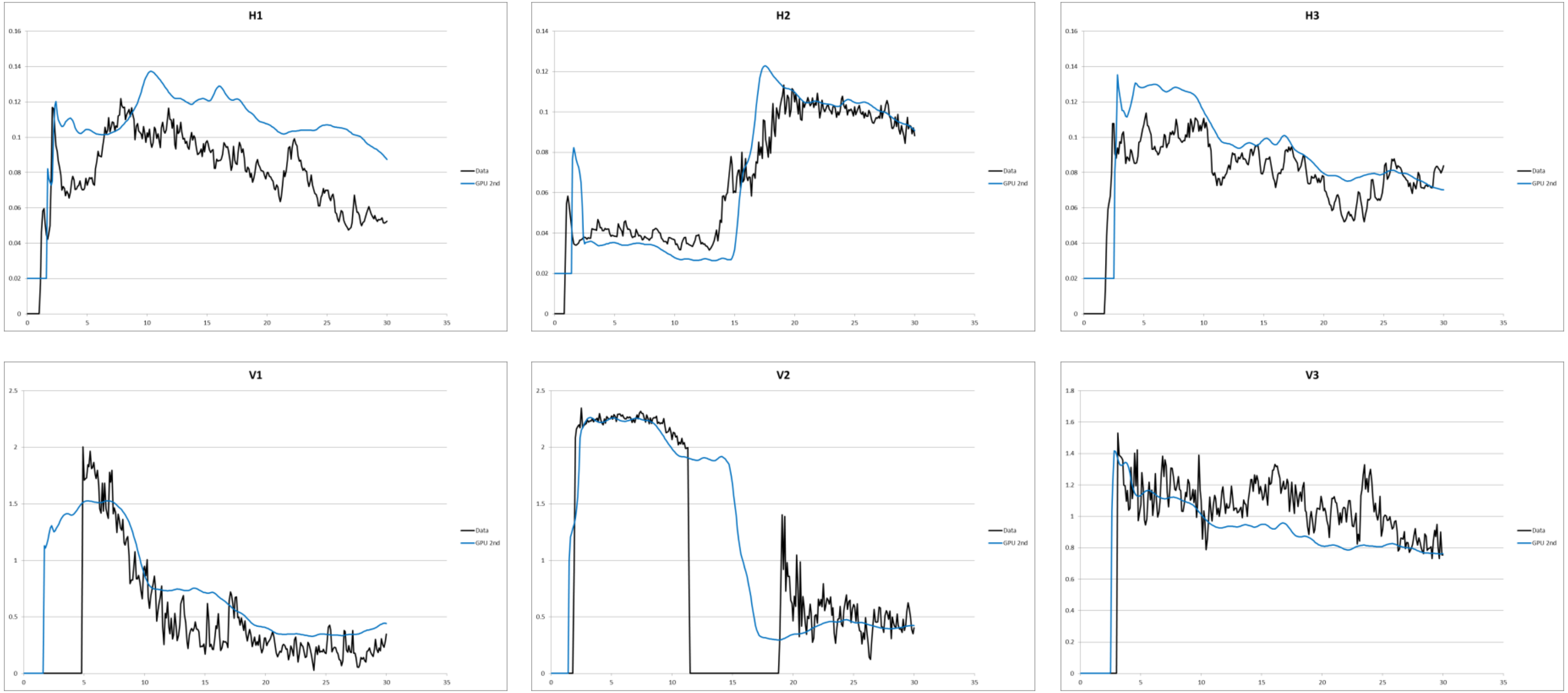
Supercritical flow

Hydraulic jumps

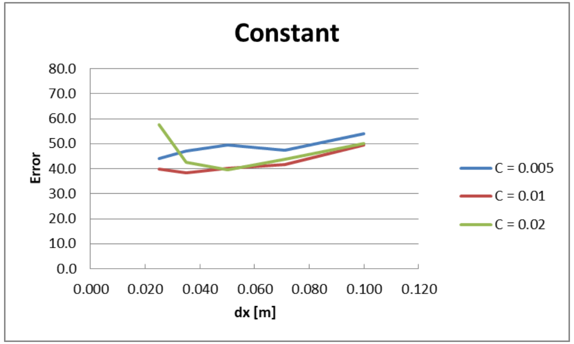


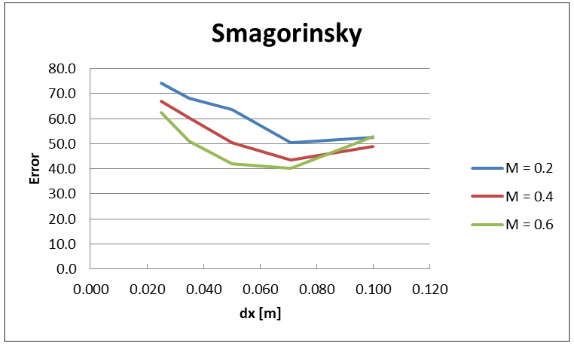


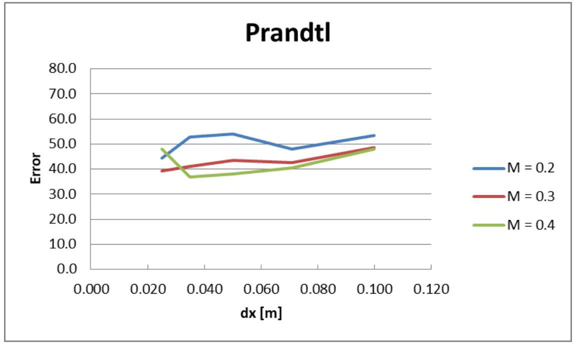
Gauge Data (first three gauges)

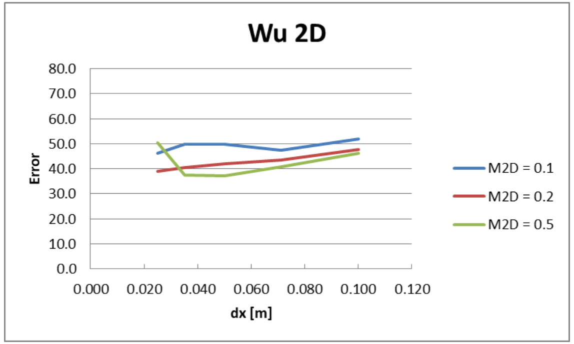


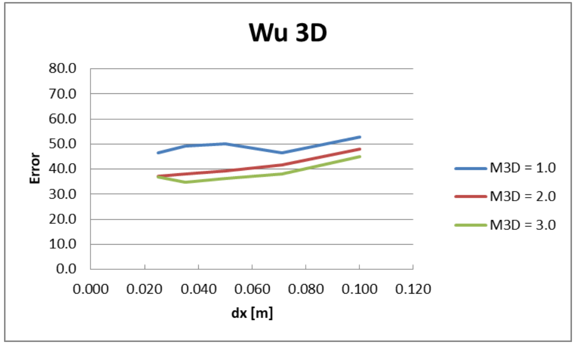
UKEA T06 Error vs Mesh Size





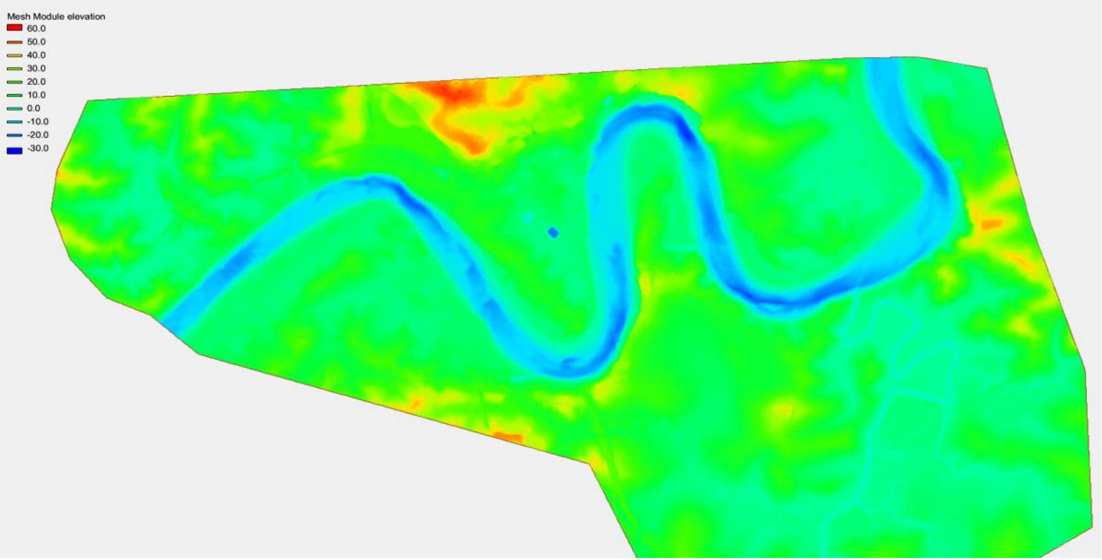




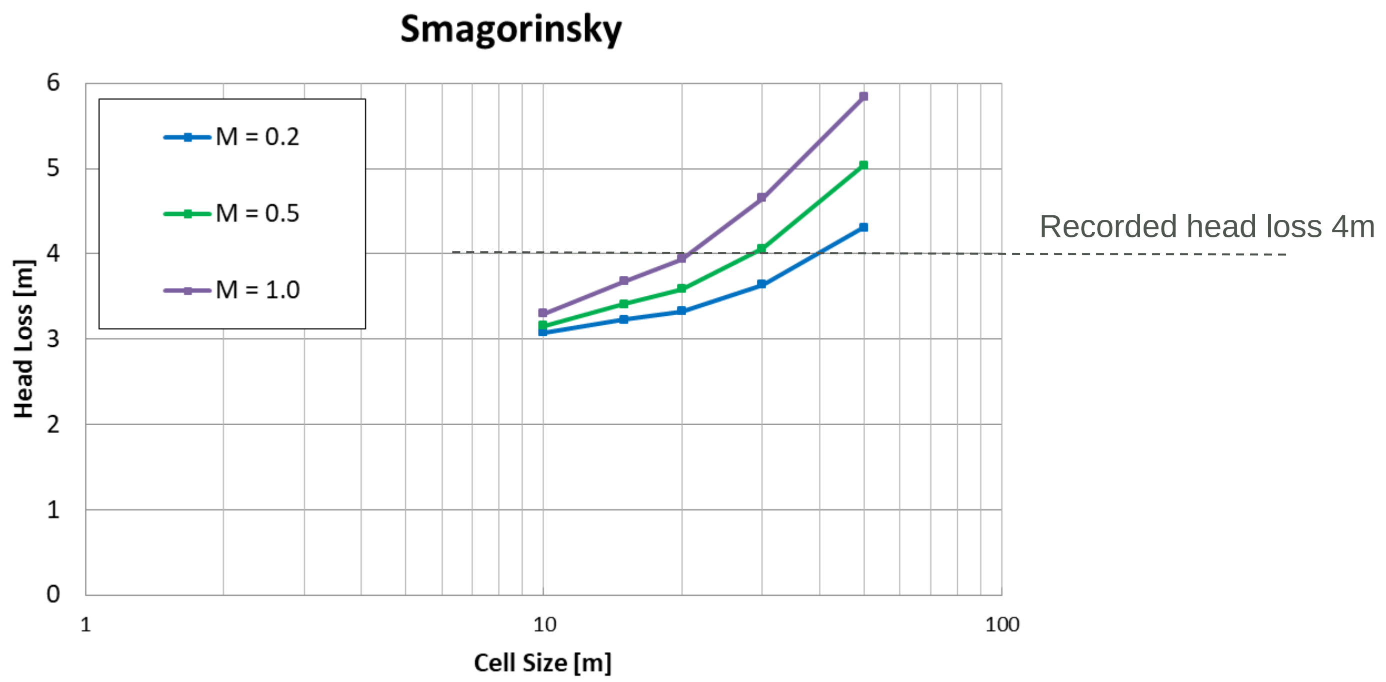


Meandering River – section of the Brisbane River

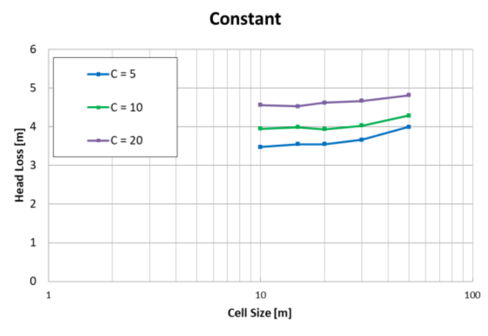
- D/S water level 2.7m
- U/S Q = 9,000 m3/s
- Steady flow model
- Peak of calibrated flow event
- Undulating bathymetry
- 20 to 30 m deep
- V_{ave} 3 to 4 m/s
- *Significant fraction of head loss due to eddy viscosity*

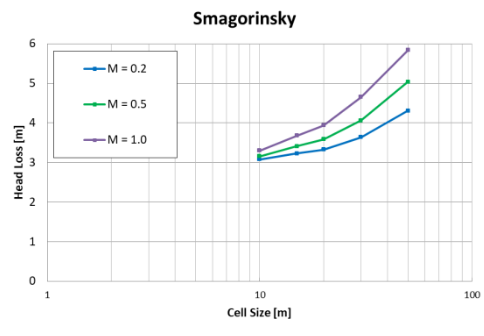


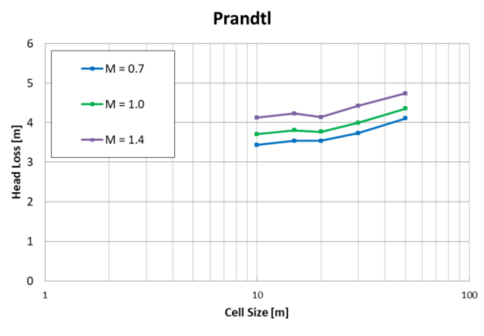
Brisbane River Head Loss vs Mesh Size

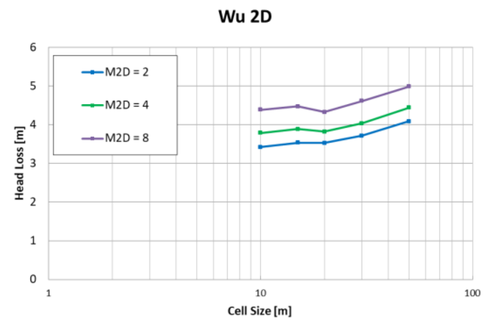


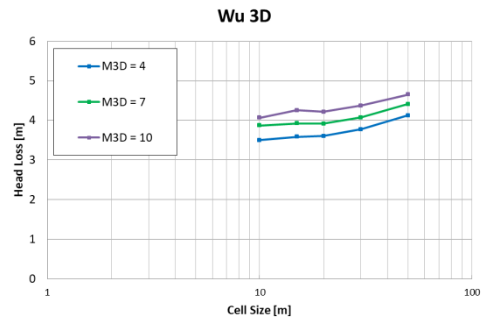
Brisbane River Head Loss vs Mesh Size











Optimum Parameters

Case	Constant	Smagorinsky	Wu 2D	Wu 3D	Prandtl
90 Deg Bend	0.004	No optimum	0.5	6	0.5
UK EA T06	0.01	No optimum	0.5	3	0.4
Brisbane River	10	No optimum	4	7	1.0

Big variation

Big variation

Requires more memory

Comfort

Lin, B., Falconer, R. A.: Tidal Flow and Transport Modelling Using ULTIMATE QUICKEST Scheme. *Journal of Hydraulic Engineering, April 1997, pp303.*

$$D_{xx} = \frac{(k_l U^2 + k_t V^2) H \sqrt{g}}{\sqrt{U^2 + V^2}\, C} + D_w; \quad D_{yy} = \frac{(k_l V^2 + k_t U^2) H \sqrt{g}}{\sqrt{U^2 + V^2}\, C} + D_w \qquad (2a,b)$$

$$D_{xy} = D_{yx} = \frac{(k_l - k_t) UVH \sqrt{g}}{\sqrt{U^2 + V^2}\, C} \qquad (2c)$$

$$C = \frac{h^{\frac{1}{6}}}{n}$$

$$D_{xx} = k_l |U| n \sqrt{g} h^{\frac{5}{6}}$$

$$\nu_{3D} = M l_m \frac{|U| n \sqrt{g}}{h^{\frac{1}{6}}}$$

So … should we have longitudinal and transverse eddy viscosity?

Summary

Constant eddy viscosity model requires scale-dependent parameter

Smagorinsky eddy viscosity model does not demonstrate mesh-size convergence

Wu 2D, Wu 3D, Prandtl models all performed well

Wu 3D showed best promise as computationally efficient *and* ‘one size fits all’

As ever, modellers encouraged to calibrate where possible, and to check mesh-size sensitivities